Budgeting for risk reduction

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ABSTRACT; Risk analysis as a convolution of hazard and consequences has been extensively used in connection with costly investments with high risk. As an extension to the determination of a risk it is useful to identify the sources of risk that can lie either on the hazard side or on the consequences' side. In the classical economical environment only a limited budget is available for a production or a public investment. If a limited budget has to be distributed among activities for risk reduction, the optimal strategy has to observe the stochastic nature of the decision problem. A solution is proposed and discussed for application in three problems: transport over a bridge, waste deposit base barrier, earthquake hazard to a lifeline.

KEYWORDS
Risk analysis; decision theory; stochastic programming; heavy weight transport; waste deposit; lifeline.

1 INTRODUCTION
Risk analysis as a convolution of hazard and consequences has been extensively used in connection with costly investments with high risk. As an extension to the determination of a risk it is useful to identify the sources of risk that can lie either on the hazard side or on the consequences' side. In the classical economical environment only a limited budget is available for a production or a public investment. Furthermore only a limited budget will be available to reduce a risk that is found not to be acceptable. The possible actions to reduce a risk are of very different nature:

1. On the hazard side it can be the reduction of an existing failure probability, where first the confidence into the failure probability as depend of stochastic parameters on the resistance side or on the load effect side has to be established.

2. On the consequences' side it can be a preparation for rapid evacuation of a possibly endangered neighbouring population or the removal of valuable goods from the respective area.

It has been shown (Klingmüller 1985, Klingmüller 1986), how the formulation of a budgeting problem for optimal risk reduction can lead to a consequent collection and structurisation of data. In refs. (Gossow and Klingmüller 1989) and (Klingmüller and Bourgund 1992) it has been extensively elaborated, how the concept can be applied to actual problems of civil engineering.

Because of the many assumptions and the fuzziness of the data applied, a risk analysis should not be finished by the definition of a risk in terms of expected losses, but the most benefit can be gained if additional evaluations are carried out to answer the following questions:

1. Are there components or elements in the analysed system that have more influence
but at the same time the reliability is increased and overall expected benefit is equal to lines A and B.

The overall benefit after the total lifetime must be assumed to be a stochastic variable because of the risk associated with the probability of failure and respective consequences.

A budget available for risk reduction can only be defined if this expected overall benefit is considered.

The risk reduction strategy has to take care of the fact, that the data on which the decision has to be taken are uncertain and given in terms of probabilities or fractiles of stochastic quantities, and additionally some activities cannot be described by a simple analytic function, but steps or edges must be described.

For the understanding of the character, first the solution of the decision problem by stochastic optimisation is demonstrated, and second for application problems suitable risk reduction functions in relation are discussed.

2 THE DECISION PROBLEM

It is assumed that a risk reduction is linearly dependent on the amount that is spend on a certain activity.

$$RR = \Sigma (a_i \cdot C_i)$$  \hspace{1cm} (1)

where $a_i$ are the stochastic coefficients of effectiveness, $C_i$ is the budget associated with activity "$i$".

To have the maximum possible risk reduction, the available total budget $C_0$ has to be distributed among the actions "$i$", as formulated by the stochastic optimisation problem:

$$\text{maximise } RR = \Sigma (a_i \cdot C_i)$$  \hspace{1cm} (2)

subject to

$$\Sigma C_i = C_0,$$

$$0 \leq C_i \leq C_0 \quad \forall \ i.$$  

As the solution of problem (2) is dependent upon the realisation of the stochastic cost coefficients, there is no unique solution. As
optimal it can be accepted to maximise the expected risk reduction. For normally-distributed stochastic cost coefficients this problem can be described by an equivalent deterministic quadratic optimisation problem by means of a utility function (Faber 1972).

To investigate the character of such a problem an example for the equivalent deterministic optimisation problem is given in two variables:

\[
\text{maximise } \bar{a}_1 \cdot C_1 + \bar{a}_2 \cdot C_2 - \frac{b}{2} (\sigma_1 \cdot C_1)^2 + (\sigma_2 \cdot C_2)^2
\]

subject to
\[
C_1 + C_2 = C_0, \\
0 \leq C_1 \leq C_0, \\
0 \leq C_2 \leq C_0.
\]

\(\bar{a}_1, \bar{a}_2\) are the mean values of the effectiveness coefficients \(a_1\) and \(a_2\), \(\sigma_1\) and \(\sigma_2\) the respective standard deviations, "b" is the "risk attitude" coefficient, that is introduced by the utility function. A high value is chosen for a high confidence towards the description of the stochastic effectiveness coefficients.

In figure 2 an example for mean values of the coefficients of effectiveness \(\bar{a}_1 = 0.2\); \(\bar{a}_2 = 1.0\) is given.

By inspection of figure 2, it can be recognised that, if the stochastic character of the effectiveness parameters is neglected, a simple linear programming problem has to be solved where the solution is given by an intersection of the linear boundaries. There is a sure optimum by dedicating the total budget towards the action "1".

For the given standard deviations, the stochastic cost coefficients are changing the objective function to quadratic, but still for low risk attitude values, i.e. risk aversion, the deterministic solution stays optimal. For a higher risk attitude value, the optimum shifts towards a partitioning of the budget to the two different actions.

In financial matters, the shown problem can be seen as associated with the well known investment strategy, i.e. to give one third of the money to a fixed interest investment, another third to buy long term state loans, and the last third to let make risky money at the stock exchange.

For general engineering problems, the situation is not as simple as given by the above example. Especially there is almost no situation with a linear benefit function. The common situation is, that the risk reduction can be either described by an exponential function or by a step-wise function (fig. 3).

The exponential function (fig. 3, R1) is appropriate if there is a high effectiveness for the initial spending, but for higher sums only a limited increase will be given, as may be the case in material testing, where the testing of samples and the installation of a sound quality assessment programme is of utmost importance. But when it comes to test all material ever used for a construction, the expenses will only lead to a stabilisation of the fourth digit in the fractile values. A step function (fig. 3, R2) is used, if a certain action is associated with a fixed sum, e.g. to add an
additional support to a bridge will cost a certain amount. To buy a half support by spending half the money is a useless action.

For this kind of general risk reduction functions problem (2) will not be linear even for fixed effectiveness coefficient. The general form of (2) will then read as

$$\max R = \sum (a_i \cdot R_i (C_i))$$

subject to

$$\sum C_i = C_0,$$

$$0 \leq C_i \leq C_0 \quad \forall i.$$

In the following examples the effectiveness coefficients are assumed as given probability values and not as stochastic variables, so that problem (4) is solved as a deterministic problem, where the expected risk reduction is maximised.

3 APPLICATION TO HEAVY WEIGHT TRANSPORT OVER A BRIDGE

In a fictitious scenario a two span bridge is to be crossed by a heavy weight transport. The risk analysis showed a risk for persons to be

$$R = (10 + 2) \cdot 1,27 \cdot 10^{-6}$$
$$= 15,24 \cdot 10^{-6}$$

where

- the 2 is referring to the driver and his companion in the truck,
- the 10 is referring to possibly endangered pedestrians,
- 1,27 \cdot 10^{-6} is a probability of failure as evaluated by structural analysis.

The material losses will not be discussed herein (cf. Klingmüller and Bourgund 1972). To reduce the risk, several activities are in competition.

1. A prohibitive system with respect to parties not concerned will cost 20 monetary units, if properly executed. For less expenditure the prohibitive system will be not as effective. And e.g. for a very low expenditure (publication in the local newspaper) the effect can be neglected. An expenditure in excess of 20 monetary units will not increase the effectiveness accordingly. At best, there will be no harm to the unconsidered parties. Thus the risk is reduced to 1,397 \cdot 10^{-5} and the risk reduction is 12,7 \cdot 10^{-6}. The associated risk reduction function is given in fig. 3.

As people may be attracted by additional activities a probability of 90 % to be fully effective will be assigned to this action.

2. A careful inspection of the bridge can be executed to have a more precise idea of the actual carrying capacity of the bridge. This activity will increase the confidence level of the failure probability, and thus the risk will be reduced by 13,76 \cdot 10^{-6} to 1,48 \cdot 10^{-5} for the spending of 30 monetary units. From zero spending onwards, a certain proportionality can be assumed between expenditure and effectiveness. Thus for this activity, the risk reduction function is assumed exponential (cf. fig. 4).

3. Increasing the strength of the mid support by injecting grouting into the soil would cost 30 monetary units. The increase in the carrying capacity would reduce the failure probability, and thus the risk will be reduced by 13,74 \cdot 10^{-6} to a new value of 1,46 \cdot 10^{-6}. As there is only an "either-or"
in this decision, a step function will be assumed (cf. fig. 3). As soil improvements are always very difficult engineering tasks, a probability of 90% to be fully effective will be assigned to this action.

4. The installation of temporary supports can be obtained at the cost of 20,000,- monetary units. With respect to total failure the strengthening only concerns one of several failure modes and therefor with respect to harm to unconcerned parties the action is assumed to reduce the risk by 4,9 \cdot 10^{-6} to a new value of 10,34 \cdot 10^{-6}. As there is only an "either-or" in this decision a step function will be assumed (cf. fig. 3). As temporarily installed supports are mostly executed by used material a probability of 98% to be fully effective will be assigned to this action.

Only four of a large number of different activities are included in this scenario, but it can be recognised, that totally different activities have to be compared, if an optimum decision is wanted. The comparison of the possible activities also shows the character of the data, that are needed for the risk reduction budgeting decision. In table 1 the data are summarised.

<table>
<thead>
<tr>
<th>action</th>
<th>cost</th>
<th>eff.</th>
<th>risk red.</th>
<th>surv. cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>20</td>
<td>0.9</td>
<td>12.7</td>
<td>0.174</td>
</tr>
<tr>
<td>(2)</td>
<td>30</td>
<td>1.0</td>
<td>13.76</td>
<td>0.218</td>
</tr>
<tr>
<td>(3)</td>
<td>30</td>
<td>0.9</td>
<td>13.74</td>
<td>0.243</td>
</tr>
<tr>
<td>(4)</td>
<td>20</td>
<td>0.98</td>
<td>4.9</td>
<td>0.41</td>
</tr>
</tbody>
</table>

The quantity "risk reduction over effectiveness times cost" is known as the survival costs and is a first indicator, where a budget is to be spend most effectively.

With respect to the more fuzzy situation of a gradual effectiveness and different shapes of risk reduction curves (see fig. 3) an optimisation problem as given by

\[
\text{maximise } R(c_1,c_2,c_3,c_4) = 0.9 \cdot RR_1(c_1) + RR_2(c_2) + 0.9 \cdot RR_3(c_3) + 0.98 \cdot RR_4(c_4)
\]

subject to
\[
c_1 + c_2 + c_3 + c_4 \leq c_0 \\
c_1 \geq 0.0, \quad c_2 \geq 0.0, \quad c_3 \geq 0.0, \quad c_4 \geq 0.0.
\]

must be solved. The solution for increasing available budget is given in table 2.

By the budget allocation as given in table 2, determined by the optimisation procedure, it can be seen, that the actions indicated by the survival costs are chosen. The quantification however of the budget allocation has to be determined by the optimisation procedure.

Figure 4: Risk reduction functions for heavy weight transport
Table 2: Budget allocation for optimal risk reduction

<table>
<thead>
<tr>
<th>action</th>
<th>available budget</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>budget allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

4 APPLICATION TO BASE BARRIER OF WASTE DEPOSIT

For the risk minimisation of waste deposits, a very complex situation of influences to the risk and actions for risk reduction has to be taken care of. In contrast to structures where mainly the ultimate limit state is guiding the design with respect to the consequences the states from complete tightness to total loss of serviceability and harm to third parties should be fuzzified in the sense that e.g. minor leakages will lead to minor consequences, This gradual decrease of serviceability is shown in figure 4.

![Figure 4: Serviceability of base barrier](image)

In addition it must be observed that the severity of consequences depend on the duration of a leakage, i.e. of the total amount of contaminating substances released and the spatial distribution in ground water and soil.

Possible actions for risk reduction are:
1. Hazard or probability of a leakage
   - increase in the thickness of a mineral clay barrier as the barrier is always built in layers of finite thickness, there a multiple step function may be chosen,
   - quality control for the clay material used with more care with soil investigations at the possible resource site monotonous risk reduction function can be appropriate
   - quality control for installation of the clay material monotonous risk reduction function can be appropriate
   - addition of one or even two layers of plastic foils to the clay barrier a step function is appropriate
   - control system for the tightness of the barrier and inspection for the installation a step function is appropriate and
   - for inspection the effectiveness will increase with the expenditure up to a certain limit.

2. Consequences
   - control of wastes to be deposed, so that the chemical property of a contaminating liquid is known and actions can be taken the effectiveness will increase with the expenditure
   - replacing of drinking water intakes, so that there is no pollution a step function may be chosen
   - additional drinking water resources as stand by redundancy a multiple step function may be chosen.

Although the actions above are chosen arbitrarily, it can be recognised, that there are more chances for risk reduction on the hazard side than on the consequences side.

5 APPLICATION TO EARTHQUAKE HAZARD TO A LIFELINE

With respect to lifelines the following hazards can be distinguished:
- hazard to supply or the resource being empty
- hazard to transport (lines)
- hazard to distribution.

A main feature of a lifeline system of high reliability is the strategic installation of active and stand-by redundancies, so that in case of failure a reduced supply can be maintained. In contrary to the situation of the waste deposit, the risk can be very much reduced by actions with reference to consequences. These may be either actions, that guarantee the most
effective distribution of a reduced supply and a layout of the lifeline system, that helps to detect local failures and supports immediate refurbishment.

In a case study the energy supply of a population of approximately 5 million people by a gas pipeline is investigated. This pipeline is leading over a distance of appr. 2000 km from a resource to the intermediate storage and distribution nodal point. A portion of less than 100 km of the pipeline is passing an area of very high seismicity.

In the scenario for risk analysis a time dependence of consequences is assumed that takes account of the fact that after failure of the pipeline the supply can be maintained for a short duration by emergency systems. After that time losses increase severely as illustrated by figure 5.

![Figure 5: Time dependent losses](image)

With such a time dependence of losses the possibilities of quick leak detection and repair become most important.

![Figure 6: Degrees of loss of serviceability](image)

For the evaluation of consequences the gradual decrease of serviceability is discretized into four steps where the degree of loss of serviceability is accepted to be directly dependent upon the earthquake peak ground acceleration (PGA, see figure 6).

In a simplified scenario four activities for risk reduction are included:

1. improved inspectability for fast leak detection
2. enhanced inspection and maintenance for lower failure probability
3. increase in wall thickness of pipe for lower failure probability
4. deviation of seismically hazardous area

In a first evaluation it has been determined that for the second activity - improved inspection and maintenance - survival cost (cost to risk reduction ratio) will be lowest. On the other hand as this activity is to be carried out over the total service life of the pipeline the overall cost in absolute monetary units will be high and might not be available. The optimal risk reduction has to be found with respect to the expected overall benefit of the pipeline that defines the available budget and by the solution of the suitable stochastic optimisation problem which leads to the budget distribution.

In a parametric study it will be investigated which amount of losses are implicitly considered in the applicable codes for buried lifelines and how an optimal design concept might be derived.

First results showed that the losses that are to be prevented by adequate design with corresponding failure probabilities in the range of $10^{-6}$ are highly in excess of the installation costs, the exact value depending on the degree of development of the nations.

Indirect consequences such as individual and/or social instability (unemployment after earthquake damage) are rated much higher and will in general justify high safety levels.

6 CONCLUSIONS

The formulation of the risk reduction problem can be seen as an important tool as to what are the important questions and which data are needed for the answer. In many situations actions to be taken for a reduction of a risk
are of totally different nature but can be combined if monetary units are assigned to them. Thus, if a risk reduction problem is formulated, the solution for different budgets will provide an optimum strategy for the distribution of the budget, but on the other hand can be utilised to reveal a lack of knowledge with respect to important data.

7. REFERENCES

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ACKNOWLEDGEMENT

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